ELSEVIER

Int. J. Therm. Sci. 41 (2002) 341–347

International Journal of Thermal Sciences

www.elsevier.com/locate/ijts

Numerical investigations and engineering applications on freezing expansion of soil restrained two-phase closed thermosyphons

Yang Pan [a,*], CunZhen Wu [b]

[a] *School of Civil Engineering and Architecture, East China Jiaotong University, Nanchang, Jiangxi, 330013, People's Republic of China*

[b] *Energy Engineering Department, Zhejiang University, Hangzhou, Zhejiang, People's Republic of China*

Abstract

A numerical simulation has been carried out to describe the coupled heat transfer of water-saturated soil with a two-phase closed thermosyphon. This problem is characterized by the phase change occurring in the water-saturated soil, moving freezing front and heat transfer of a two-phase closed thermosyphon. According to the governing equations in the porous media and heat transfer characteristics of the two-phase closed thermosyphon, the temperature distributions in the water-saturated soil and the moving freezing front are solved numerically by the finite-difference method. The predictions of the present study are well agreed with the measured data. The mechanism of the freezing expansion restrained by the two-phase closed thermosyphon is exposed, based on which the effective radius can be determined for engineering applications. © 2002 Éditions scientifiques et médicales Elsevier SAS. All rights reserved.

Keywords: Soil freezing expansion; Freezing front; Two-phase closed thermosyphon; Coupled heat transfer; Temperature field

1. Introduction

In the cold region, some construction foundations are often deformed and damaged due to freezing expansion of soil in winter. One of the ways to prevent the foundation damage is use of two-phase closed two-phase closed thermosyphons in which there are two sections, the evaporator section buried underground and the condenser section exposed in air. When the air temperature is below the soil temperature, the working liquid in the evaporator section of the two-phase closed thermosyphon will be evaporated, and the vapor rises to the condenser section then is condensed by chill air. In this way, since the thermal energy in the soil is transferred and dissipated into the environment by the two-phase closed thermosyphon, the soil temperature distribution is significantly changed.

Numerical studies on freezing heat transfer in water-saturated porous media have been extensively investigated [1–3]. Wu and Gu [4,5] experimentally and numerically investigated solidification and freezing with a two-phase closed thermosyphon, considering the coupled heat transfer between the thermosyphon and pure water, and moving freezing front. In the freezing heat transfer with two-phase closed thermosyphon, heat transfer in the two-phase closed thermosyphon, especially heat transfer in the evaporator section, has significant effect on the freezing heat transfer. The boiling heat transfer in the evaporator is usually divided into two boiling zones, namely, liquid film boiling and liquid pool boiling. Some correlations [7–9] were proposed for the falling film boiling and the pool boiling, including the pool boiling height and boiling heat transfer coefficients.

However, for the coupled heat transfer of a two-phase closed thermosyphon with the water-saturated soil, there was little information to be presented yet. One reason is a new technology and complex problem in the engineering application. Another difficulty lies in the presence of unsteady heat transfer and flow in water-saturated porous media, phase change in porous media, moving boundary layer, and the thermal interaction between porous media and the two-phase closed thermosyphon.

The primary purpose of the present work is to analyze and explore the mechanism of soil frreezing expansion restrained by the two-phase closed thermosyphon, by describing this problem mathematically and solving the model numerically. In what follows, a mathematical model, including heat transfer and flow in the water-saturated soil, heat transfer in the two-phase closed thermosyphon, and coupled heat transfer phenomenon between soil and the two-phase closed

* Correspondence and reprints.
E-mail addresses: hdjdpan@sina.com (Y. Pan), czwu@eed.zju.edu.cn (C.Z. Wu).

1290-0729/02/$ – see front matter © 2002 Éditions scientifiques et médicales Elsevier SAS. All rights reserved.
PII: S1290-0729(02)01324-8

Nomenclature

A	area	m^2
B	coefficients in Eqs. (5a) and (5c)	
C	defined in Eq. (5e)	
c	specific heat	$J\cdot kg^{-1}\cdot °K^{-1}$
d	diameterm	
f	local liquid fraction	
F	defined in Eq. (9c)	
g	gravitational acceleration	
H_p	boiling liquid pool height	m
H_l^*	filling ratio of working liquid	
H_p^*	dimensionless boiling liquid pool height	
h	heat transfer coefficient	$W\cdot m^{-2}\cdot K^{-1}$
Δh	latent heat of solidification	$J\cdot kg^{-2}$
K	permeability of porous media	
k	thermal conductivity	$W\cdot m^{-1}\cdot K^{-1}$
L	length	m
p	pressure	Pa
Pr	Prandtl number	
Q	heat transfer rate	W
q	heat flux	$W\cdot m^{-2}$
R	radius	m
r	radius coordinate	
Re	Reynolds number	
S	source term	
T	temperature	K or °C
u, v	velocity in z and r direction	$m\cdot s^{-1}$
V	volume	m^3
z	axial coordinate	

Greek symbols

τ	time	h
Δ	difference	
λ	latent heat of vaporization	$J\cdot kg^{-1}$
ρ	density	$kg\cdot m^{-3}$
ε	porosity	%
μ	dynamic viscosity	$kg\cdot m^{-1}\cdot s^{-1}$
σ	surface tension	$N\cdot m^{-1}$
ν	kinematic viscosity	$m^2\cdot s^{-1}$
δ	thickness	m

Subscripts

a	air
b	boundary
c	condensation
e	evaporation or east nodal
f	liquid film or at fusion
i	inner
m, max	average and maximum
n	north nodal
o	outer
p	porous media or liquid pool, curent nodal
r	r-direction
s	saturated or south nodal
t	heat pipe
v	vapor
w	wall or west nodal
z	z-direction
nb	nearby nodal
eff, f	effective

Superscripts

o	old value
*	dimensionless
¯	average

thermosyphon, is developed. The governing equations are solved by the finite difference method. Then the mechanism on the soil expansion restrained by the thermosyophon is expressed analytically. Based on the mechanism and the calculated results, the effective radius which is an important parameter for the engineering application, is proposed.

2. Analysis and model

The physical model, in this study, is a two-dimensional cylindrical coordinate system, as depicted in Fig. 1. For the soil, the following assumptions are made:

(1) The soil is a porous media with isotropic and homogeneous;
(2) The fluid flow in unfrozen porous layer is laminar and incompressible;
(3) The volume change resulted from the frozen expansion can be negligible and the particles of the soil are rigid;
(4) The thermophysical properties are constant with the exception of the density in the buoyance force term;
(5) The effect of the foundation on the fluid flow and soil heat transfer is not considered.

According to the above assumptions, conservation of mass, momentum and thermal energy in the water-saturated soil yields the governing equations:

$$\frac{\partial(ru)}{\partial z}+\frac{\partial(rv)}{\partial r}=0 \tag{1}$$

$$\frac{\rho_l}{\varepsilon}\frac{\partial u}{\partial \tau}+\frac{\rho_l}{\varepsilon^2}\left(u\frac{\partial u}{\partial z}+v\frac{\partial u}{\partial r}\right)=-\frac{\partial p}{\partial z}+\frac{\mu_l}{\varepsilon}\left(\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 u}{\partial r^2}\right)+S_z+S_\mathrm{b} \tag{2}$$

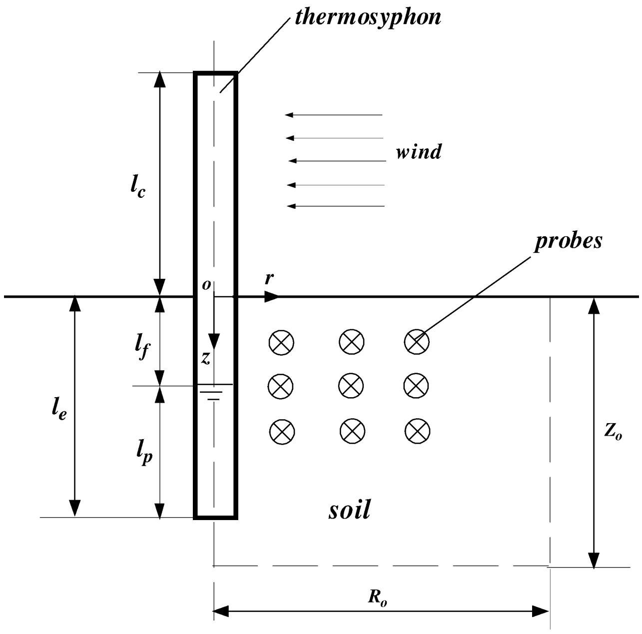

Fig. 1. Schematic diagram.

$$\frac{\rho_l}{\varepsilon}\frac{\partial v}{\partial \tau}+\frac{\rho_l}{\varepsilon^2}\left(u\frac{\partial v}{\partial z}+v\frac{\partial v}{\partial r}\right) = -\frac{\partial p}{\partial r}+\frac{\mu_l}{\varepsilon}\left(\frac{\partial^2 v}{\partial z^2}+\frac{\partial^2 v}{\partial r^2}\right)+S_r \tag{3}$$

$$\overline{\rho c}\frac{\partial T}{\partial \tau}+\varepsilon\rho_l\left(u\frac{\partial T}{\partial z}+v\frac{\partial T}{\partial r}\right) = k_{\text{eff}}\left(\frac{\partial^2 T}{\partial z^2}+\frac{\partial^2 T}{\partial r^2}\right)+\varepsilon\rho_l\Delta h\frac{\partial f}{\partial \tau} \tag{4}$$

where

$$S_z=-\left(B+\frac{\mu_l}{K}+\frac{\rho_l C|u|}{K}\right)u \tag{5a}$$

$$S_{\text{b}}=-\rho_{\text{m}} g\omega\left(|T_{\text{f}}-T_{\text{m}}|^x-|T-T_{\text{m}}|^x\right) \tag{5b}$$

$$S_r=-\left(B+\frac{\mu_l}{K}+\frac{\rho_l C|v|}{K}\right)v \tag{5c}$$

and

$$K=\frac{d^2\varepsilon^2}{150(1-\varepsilon)^2} \tag{5d}$$

$$C=\frac{1.75d}{(150\varepsilon^2)^{0.5}} \tag{5e}$$

$$\overline{\rho c}=\varepsilon\left[f\rho_l c_l+(1-f)\rho_{\text{s}} c_{\text{s}}\right]+(1-\varepsilon)\rho_{\text{p}} c_{\text{p}} \tag{5f}$$

$$k_{\text{eff}}=\varepsilon\left[f k_l+(1-f)k_{\text{s}}\right]+(1-\varepsilon)k_{\text{p}} \tag{5g}$$

where B, ρ_{m}, T_{m}, x, and ω are given by Gebhort and Mollendort [6]

$$B=(1-f)\times 10^9$$

$$\rho_{\text{m}}=999.972\ \text{kg·m}^{-3}$$

$$T_{\text{m}}=4.029325$$

$$x=1.894816$$

$$\omega=9.297173\times 10^{-6}$$

For the two-phase closed thermosyphon, the following assumptions are made:

(1) The liquid film inside the two-phase closed thermosyphon is laminar and the effect of the vapor flow can be negligible;
(2) The vapor in the two-phase closed thermosyphon is saturated and its temperature is uniform;
(3) The evaporator section of the two-phase closed thermosyphon consists of two parts, the liquid film part and the boiling pool part, the film temperature in the liquid film part equaling the vapor temperature, while the temperature in the boiling pool part being the saturation temperature which varies along its height due to the hydrostatic pressure.

The heat transfer coefficients for the condensation, film evaporator and pool boiling are written respectively as follows [8,9].

For condensation in the thermosyphon, one has

$$h_{\text{c}}\frac{(\nu_l^2/g)^{1/3}}{k_l}=\left(\frac{4}{3}\right)^{4/3}Re_{\text{c}}^{-1/3} \tag{6a}$$

where

$$Re_{\text{c}}=\frac{4q_{\text{c}}L_{\text{c}}}{\lambda\mu_l} \tag{6b}$$

For the falling film boiling, the coefficient is written as

$$h_{\text{f}}\frac{(\nu_l^2/g)^{1/3}}{k_l}=\left(\frac{4}{3}\right)^{4/3}Re_{\text{f}}(z)^{-1/3} \tag{7a}$$

where

$$Re_{\text{f}}(z)=\frac{4q_{\text{e}}(L_{\text{e}}-z)}{\lambda\mu_l} \tag{7b}$$

For the pool boiling, one has

$$\frac{h_{\text{p}}d_{\text{i}}}{k_l}=14.45\left(\frac{d_{\text{i}}q_{\text{e}}}{\lambda\mu_l}\right)^{0.39}Pr^{0.75}\left(\frac{\rho_l}{\rho_{\text{v}}}\right)^{0.2}\left(\frac{H_{\text{p}}}{d_{\text{i}}}\right)^{0.12} \tag{8}$$

where the dimensionless boiling pool height is given by

$$H_{\text{p}}^*=\frac{H_{\text{p}}}{L_{\text{e}}}=\left(1.59F^{0.95}Re_{\text{f}}^{-0.35}H_l^{*0.37}+1\right)H_l^* \tag{9a}$$

$$H_l^*=\frac{L_{\text{p}}}{L_{\text{e}}} \tag{9b}$$

$$F=\frac{4q_{\text{e}}L_{\text{e}}}{\lambda\rho_{\text{v}}}\Big/\left\{1.53d_{\text{i}}\left[\sigma g(\rho_l-\rho_{\text{v}})/\rho_{\text{v}}^2\right]^{0.25}\right\} \tag{9c}$$

$$Re_{\text{f}}=\frac{4q_{\text{e}}L_{\text{e}}}{\lambda\mu_l} \tag{9d}$$

It is noted that L_{p} is different from H_{p}. The former denotes the filling liquid pool height at the static state, and the latter is the boiling liquid pool height under the operating state.

The conjugation of heat transfer between the two-phase closed thermosyphon and the porous media is described by the thermal balance, and is written by

$$k_{\mathrm{eff}} \frac{\partial T}{\partial r}\bigg|_{r=R_{\mathrm{i}}} = h_{\mathrm{e}}(T_{\mathrm{s}} - T_{\mathrm{wei}}) \tag{10}$$

where

$$h_{\mathrm{e}} = \begin{cases} h_{\mathrm{f}} & \text{for liquid film} \\ h_{\mathrm{p}} & \text{for liquid pool} \end{cases} \tag{11}$$

Notice that the outer wall temperature of the evaporator section T_{weo} is a major parameter that is an interface of the coupled heat transfer as well as the boundary conditions in soil domain. T_{weo} in the liquid pool is different along the axial direction due to the static pressure. The outer wall temperature distributions in the liquid pool is given by

$$T_{\mathrm{weo}}(z) = T_{\mathrm{s}}(z) + q_{\mathrm{e}}\left(\frac{\delta_{\mathrm{w}} A_{\mathrm{ei}}}{k_{\mathrm{w}} A_{\mathrm{em}}} + \frac{1}{h_{\mathrm{e}}}\right) \tag{12}$$

where

$$T_{\mathrm{s}}(z) = \begin{cases} T_{\mathrm{s}} & \text{for liquid film} \\ f\big[P(T_{\mathrm{s}}) + \rho_l g(z - L_{\mathrm{e}} - H_{\mathrm{p}})\big] & \text{for liquid pool} \end{cases} \tag{13}$$

Since the condenser section is finned, the external convection heat transfer coefficient is written by the Brigg's correlation,

$$h_{\mathrm{a}} = \frac{k_{\mathrm{a}}}{d_{\mathrm{o}}} 0.1378\, Re_{\mathrm{a}}^{0.718}\, Pr_{\mathrm{a}}^{0.333} \left(\frac{s_{\mathrm{f}}}{l_{\mathrm{f}}}\right)^{0.296}$$

Theoretically, the convective heat rate is equal to the heat rate released from soil.

The initial values and boundary conditions for the entire domain are:

$$\tau = 0: \quad T = T(z) \tag{14a}$$

$$u = v = 0 \tag{14b}$$

$$z = 0: \quad k_{\mathrm{eff}} \frac{\partial T}{\partial z} = h_{\mathrm{ga}}(T - T_{\mathrm{a}}) \tag{15a}$$

$$u = v = 0 \tag{15b}$$

$$z = \infty: \quad T = T(\tau) \tag{16a}$$

$$u = 0 \tag{16b}$$

$$z > L_{\mathrm{e}} \quad \text{and} \quad r = 0: \quad \frac{\partial T}{\partial r} = 0 \tag{17a}$$

$$v = 0 \tag{17b}$$

$$z \leqslant L_{\mathrm{e}} \quad \text{and} \quad r = R_{\mathrm{i}}: \quad T = T_{\mathrm{weo}}(z) \tag{18a}$$

$$u = v = 0 \tag{18b}$$

$$r = \infty: \quad \frac{\partial T}{\partial r} = 0 \tag{19a}$$

$$v = 0 \tag{19b}$$

The above equations are solved by the SIMPLE algorithm [10].

In the energy conservation equation, the local liquid fraction of current time step in each control volume must be updated by that of last time step [11].

$$f = f^{\circ} + \left(\sum a_{\mathrm{nb}} T_{\mathrm{nb}} + a_{\mathrm{p}}^{\circ} T_{\mathrm{p}}^{\circ} - a_{\mathrm{p}} T_{\mathrm{p}}\right)\left[\frac{\varepsilon c_l \Delta\tau}{(\rho_l \Delta h \Delta V)}\right] \tag{20}$$

If $f < 0$ then $f = 0$, and if $f > 1$ then $f = 1$. If $0 < f < 1$ set $T_{\mathrm{p}} = T_{\mathrm{f}}$.

The equations are iteratively solved by the finite difference scheme. Calculation begins from the two-phase closed thermosyphon. First, a wall temperature of the evaporator section of the two-phase closed thermosyphon is evaluated by assuming heat transfer rate of the two-phase closed thermosyphon, or using the last calculated one. Then, in the light of the boundary conditions and the calculated wall temperature of the evaporator, the temperature field and the stream function relevant to Eqs. (1)–(3) are gained, respectively. The heat rate of the two-phase closed thermosyphon is renewed. Finally, the iterations are repeated until the requirement convergence criterion is satisfied:

$$\left|\frac{Q_r - Q_{\mathrm{b}}}{Q_r}\right| < 10^{-3} \tag{21}$$

where

$$Q_r = \int_0^{z_{\mathrm{o}}} \int_0^{R_{\mathrm{o}}} \big[\rho c\big(T - T^{\circ}\big) + \varepsilon\rho\Delta h\big(f - f^{\circ}\big)\big] r \,\mathrm{d}r \,\mathrm{d}z \tag{22}$$

$$Q_{\mathrm{b}} = Q_{\mathrm{w}} + Q_{\mathrm{e}} + Q_{\mathrm{s}} + Q_{\mathrm{n}} \tag{23}$$

Eq. (21) is a system error based on energy balance equation. Q_r denotes the enthalpy increase of the control volume caused by the change of temperature, local liquid fraction and phase change. Q_{b} is the total energy exchange through all boundary surfaces during $\mathrm{d}\tau$. Q_r should equal Q_{b}.

3. Measurement of soil temperature

In the present work, all soil temperature values are measured on the engineering spot. The measurement system is simply composed of a two-phase closed thermosyphon which is made from the working liquid of R21 and the carbon-steel tube with a 0.0054 m I.D. and length of 3.7 m, and a number of measuring probes.

The measurement is divided into two groups with the same radial and deep embedded points to eliminate measuring error, as shown in Fig. 2. Each group consists of thirty measuring points. Five different radial positions are set, and $\boldsymbol{r}$ equals 0.5, 0.75, 1.2, 1.5 and 2.0 m respectively along the radial direction of the thermosyphon. Each radial position involves six different depths, z is equal to 0.5, 1.0, 1.5, 2.0 2.5 and 3.0 m, respectively.

The thermistor sensors are used to measure soil temperature. The measured values are treated in terms of mean-value of two groups with the same radial and deep position.

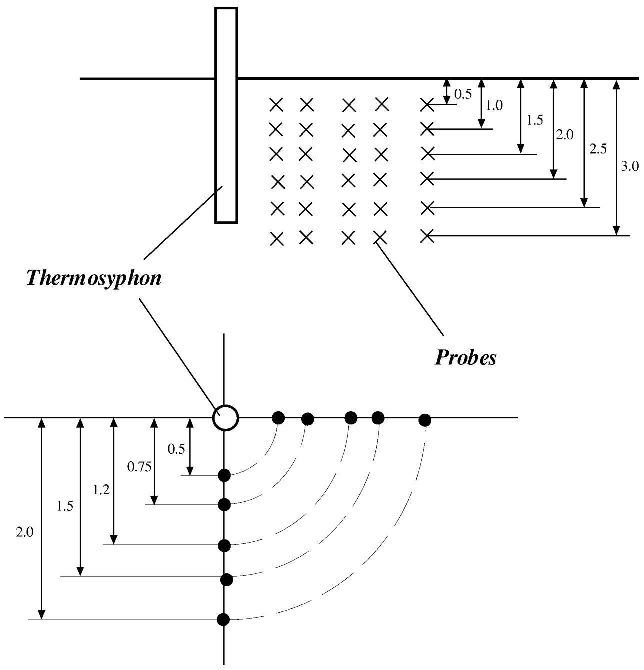

Fig. 2. Scheme of arrangement of measuring points.

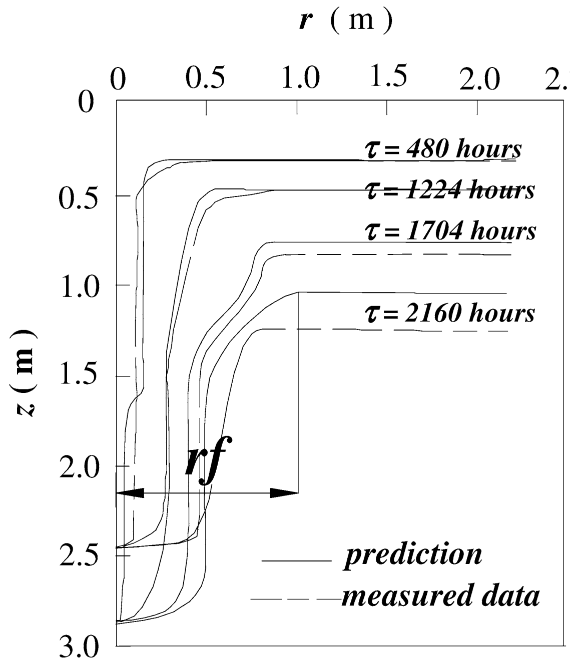

Fig. 3. Variation of the freezing front.

4. Results and discussions

In accordance with the porosity of the soil, meteorologic conditions and other data in the cold region of the northeast China, some parameters, as shown in Table 1, are listed. The total calculating time is from November 8th of 1996 to February 6th of 1997 which is called the soil freezing period. To observe the moving regulation of the soil freezing front, it is defined that soil freezes when the soil temperature is less than or equal to 0 °C and a region of frozen soil is called the frozen region. The interface between the frozen region and unfrozen region is known as the freezing front.

Fig. 3 displays the moving regulation of the freezing front predicted by the present work as well as measured by experiment. With the time, the frozen region is thickened not only near the ground surface but also near the two-phase closed thermosyphon. While only the frozen region exists near the ground surface when the two-phase closed thermosyphons are absent. In the other words, after using the two-phase closed thermosyphons, the range of frozen region extents along the deep direction. Meanwhile the frozen region, like a reinforcedrib, can restrain the soil expansion so as to prevent the foundation from damage.

After imbedding the two-phase closed thermosyphon, the local soil temperature distribution is shown in Fig. 4. For the same deepness z, the change of temperature is strongly dependent on the radial distance r out of the two-phase closed thermosyphon, which differs from the case without the two-phase closed thermosyphon in which the temperature field is horizontal isothermal. The freezing expansion always occurs in the temperature region from 0° to 4° for the water-saturated soil, and this temperature region is considered as the dangerous temperature region. If this range is much wide at a certain time, a strong expansion force will be formed such that the foundation is damaged. It is seen that the range of the dangerous temperature region is reduced at any time so that the destructive strength that results from the soil freezing expansion is weakened. This is a major mechanism of the two-phase closed thermosyphons restraining the soil freezing expansion. Figs. 3 and 4 also illustrate the predictions of the present study are in good agreement with the measured data due to the presence of the thermosyphon.

Table 1
Some parameters

Parameter	Value	Parameter	Value
Evaporator length, L_e	2.7 m	Inner diameter of pipe, d_i	0.054 m
Condenser length, L_c	1.0 m	Convective coefficient, h_{ga}	8.5 W·m^{-2}·K^{-1}
Fin height of condenser, L_{f2}	0.05 m	Average wind velocity, U_a	2.89 m·s^{-1}
Fin space of condenser, s_f	0.025 m	Particle diameter of the soil, d	0.002 m
Fin thickness, δ_f	0.003 m	Porosity of the soil, ε	51.1%
Outer diameter of pipe, d_o	0.06 m	Calculating time step, $\Delta\tau$	6 h

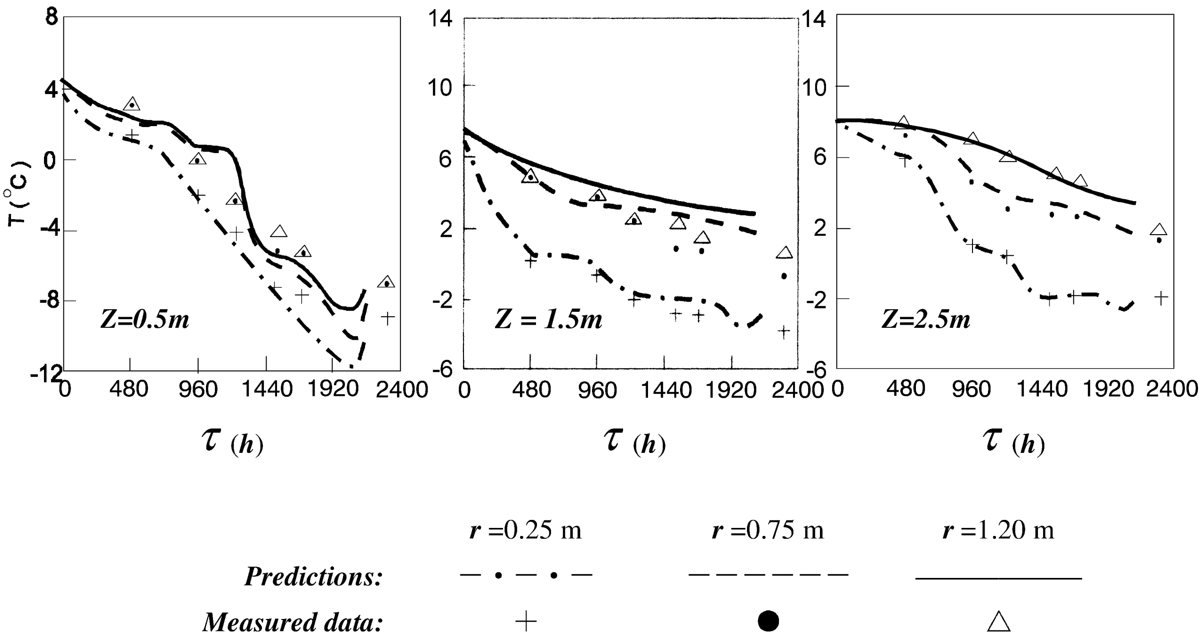

Fig. 4. Change of soil temperature field.

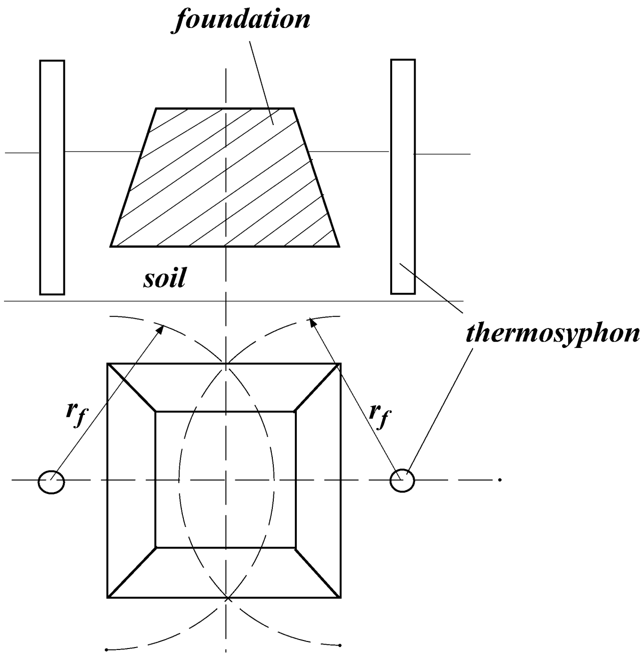

Fig. 5. Scheme of arrangement of the thermosyphons.

However, the effect of the two-phase closed thermosyphon on the soil temperature is limited. It is a key problem how to design and arrange the two-phase closed thermosyphons reasonably and efficiently for engineering application. According to the calculated results, r_f, the maximum frozen radius along the radial direction of the two-phase closed thermosyphon on February 6th is taken as an effective radius. Generally speaking, it is required that the effective radius of the two-phase closed thermosyphons should cover the horizontal profile of a foundation as much as possible for the engineering design, as shown in Fig. 5. In this way, the change range of the soil temperature field will be maximum, and a small soil expansion range will be controlled at any time. Numbers of the two-phase closed thermosyphon, space between two two-phase closed thermosyphons and between the two-phase closed thermosyphon and the foundation can be determined by using the effective radius such that a optimal control of the soil freezing expansion can be realized.

5. Conclusion

Based on topographical characteristics and meteorologic conditions, the model on coupled heat transfer between the two-phase closed thermosyphon and water-saturated soil is carried out. The prediction of the present work is in good agreement with the data measured. Analytically, the major mechanism of the two-phase closed thermosyphon controlling the soil freezing expansion is:

(1) The two-phase closed thermosyphon changes the soil temperature field so that the freezing expansion range is reduced;
(2) A steady construction formed by the frozen region restrains the destructive force that results from the freezing expansion. The effective radius of the two-phase closed thermosyphon r_f is a theoretical secundum for engineering application.

References

[1] C. Bechermann, R. Viskanta, Natural convection solid/liquid phase change in porous media, Internat. J. Heat Transfer 31 (1988) 33–46.

[2] A. Sasaki, S. Aliba, S. Fukusako, Transient freezing heat transfer in a water-saturated porous media, Proc. of 2nd Internat. Sympos. On Cold Regions Heat Transfer, Canada (1989) 291–296.
[3] A. Sasaki, S. Aliba, S. Fukusako, Numerical study on freezing heat transfer in water-saturated porous media, Numer. Heat Transfer A 18 (1990) 17–32.
[4] C.Z. Wu, W.B. Gu, Y.C. Hu, Coupled heat transfer of a two-phase closed thermosyphon with solidification, in: Proc. of 2nd Internat. Sympos. on Cold Regions Heat Transfer, Canada, 1989, pp. 183–188.
[5] C.Z. Wu, W.B. Gu, Heat transfer prediction of two-phase closed thermosyphon with freezing, in: Proc. of 4th Internat. Sympos. On Cold Regions Heat Transfer, Canada, 1993, pp. 291–297.
[6] B. Gebhart, J. Mollendort, A new density relation for pure and saline water, Deep Sea Res. 24 (1977) 831–848.
[7] M. Shiraish, K. Kikuchi, T. Yamnish, in: Investigation of Heat Transfer Characteristics of a Two-phase Closed Thermosyphon, Advances in Heat Pipe Technology, Pergamon, 1982, pp. 95–104.
[8] Jialun He, Tongze Ma, Zhengfang Zhang, Heat transfer characteristics in the evaporator section of a two-phase closed thermosyphon, in: Proc. of 8th Internat. Heat Pipe Conf., Beijing, China, 1992, BP-6.
[9] Jialun He, Tongze Ma, Zhengfang Zhang, Investigation of boiling pool height of a two-phase closed thermosyphon, in: Proc. of 8th Internat. Heat Pipe Conf., Beijing, China, 1992, B-24.
[10] S.V. Patankar, Numerical Heat Transfer and Fluid Flow, Hemisphere, Washington DC, 1980.
[11] M. Lacroix, V.R. Voller, Finite difference solution of solidification phase change problem: Transformed versus fixed grids, Numer. Heat Transfer B 17 (1990) 25–41.